

Problem 2.88

[Difficulty: 3]

2.88 A supersonic aircraft travels at 2700 km/hr at an altitude of 27 km. What is the Mach number of the aircraft? At what approximate distance measured from the leading edge of the aircraft's wing does the boundary layer change from laminar to turbulent?

Given: Data on supersonic aircraft

Find: Mach number; Point at which boundary layer becomes turbulent

Solution:

Basic equation $V = M \cdot c$ and $c = \sqrt{k \cdot R \cdot T}$ For air at STP, $k = 1.40$ and $R = 286.9 \text{ J/kg} \cdot \text{K}$ (53.33 ft.lbf/lbm $^\circ\text{R}$).

Hence
$$M = \frac{V}{c} = \frac{V}{\sqrt{k \cdot R \cdot T}}$$

At 27 km the temperature is approximately (from Table A.3) $T = 223.5 \cdot \text{K}$

$$M = \left(2700 \times 10^3 \cdot \frac{\text{m}}{\text{hr}} \times \frac{1 \cdot \text{hr}}{3600 \cdot \text{s}} \right) \cdot \left(\frac{1}{1.4} \times \frac{1}{286.9} \cdot \frac{\text{kg} \cdot \text{K}}{\text{N} \cdot \text{m}} \times \frac{1 \cdot \text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \times \frac{1}{223.5} \cdot \frac{1}{\text{K}} \right)^{\frac{1}{2}} \quad M = 2.5$$

For boundary layer transition, from Section 2-6 $Re_{\text{trans}} = 500000$

Then
$$Re_{\text{trans}} = \frac{\rho \cdot V \cdot x_{\text{trans}}}{\mu} \quad \text{so} \quad x_{\text{trans}} = \frac{\mu \cdot Re_{\text{trans}}}{\rho \cdot V}$$

We need to find the viscosity and density at this altitude and pressure. The viscosity depends on temperature only, but at 223.5 K = -50°C, it is off scale of Fig. A.3. Instead we need to use formulas as in Appendix A

At this altitude the density is (Table A.3) $\rho = 0.02422 \times 1.225 \frac{\text{kg}}{\text{m}^3} \quad \rho = 0.0297 \frac{\text{kg}}{\text{m}^3}$

For μ
$$\mu = \frac{b \cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}} \quad \text{where} \quad b = 1.458 \times 10^{-6} \cdot \frac{\text{kg}}{\text{m} \cdot \text{s} \cdot \text{K}^{\frac{1}{2}}} \quad S = 110.4 \cdot \text{K}$$

$$\mu = 1.459 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}} \quad \mu = 1.459 \times 10^{-5} \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$$

Hence
$$x_{\text{trans}} = 1.459 \times 10^{-5} \cdot \frac{\text{kg}}{\text{m} \cdot \text{s}} \times 500000 \times \frac{1}{0.0297} \cdot \frac{\text{m}^3}{\text{kg}} \times \frac{1}{2700} \times \frac{1}{10^3} \cdot \frac{\text{hr}}{\text{m}} \times \frac{3600 \text{ s}}{1 \cdot \text{hr}} \quad x_{\text{trans}} = 0.327 \text{ m}$$